## Exercise 1G: Solutions

**1 a**
$$\frac{2x}{3}+\frac{3x}{2}=\frac{4x+9x}{6}$$
$$=\frac{13x}{6}$$

**b**
$$\frac{3a}{2}-\frac{a}{4}=\frac{6a-a}{4}$$
$$=\frac{5a}{4}$$

**c**
$$\frac{3h}{4}+\frac{5h}{8}-\frac{3h}{2}=\frac{6h+5h-12h}{8}$$
$$=-\frac{h}{8}$$

**d**
$$\frac{3x}{4}-\frac{y}{6}-\frac{x}{3}=\frac{9x-2y-4x}{12}$$
$$=\frac{5x-2y}{12}$$

**e**
$$\frac{3}{x}+\frac{2}{y}=\frac{3y+2x}{xy}$$

**f**
$$\frac{5}{x-1}+\frac{2}{x}=\frac{5x+2(x-1)}{x(x-1)}$$
$$=\frac{5x+2x-2}{x(x-1)}$$
$$=\frac{7x-2}{x(x-1)}$$

**g**
$$\frac{3}{x-2}+\frac{2}{x+1}=\frac{3(x+1)+2(x-2)}{(x-2)(x+1)}$$
$$=\frac{3x+3+2x-4}{(x-2)(x+1)}$$
$$=\frac{5x-1}{(x-2)(x+1)}$$

**h**
$$\frac{2x}{x+3}-\frac{4x}{x-3}-\frac{3}{2}=\frac{4x(x-3)-8x(x+3)-3(x+3)(x-3)}{2(x+3)(x-3)}$$
$$=\frac{4x^2-12x-8x^2-24x-3(x^2-9)}{2(x+3)(x-3)}$$
$$=\frac{4x^2-12x-8x^2-24x-3x^2+27}{2(x+3)(x-3)}$$
$$=\frac{-7x^2-36x+27}{2(x+3)(x-3)}$$

**i**
$$\frac{4}{x+1}+\frac{3}{(x+1)^2}=\frac{4(x+1)+3}{(x+1)^2}$$
$$=\frac{4x+4+3}{(x+1)^2}$$
$$=\frac{4x+7}{(x+1)^2}$$

**j**
$$\frac{a-2}{a}+\frac{a}{4}+\frac{3a}{8}=\frac{8(a-2)+2a^2+3a^2}{8a}$$
$$=\frac{5a^2+8a-16}{8a}$$

**k** $$\begin{aligned} 2x - \frac{6x^2-4}{5x} &= \frac{10x^2-(6x^2-4)}{5x} \\ &= \frac{10x^2-6x^2+4}{5x} \\ &= \frac{4x^2+4}{5x} \\ &= \frac{4(x^2+1)}{5x} \end{aligned}$$

**l** $$\begin{aligned} \frac{2}{x+4} - \frac{3}{x^2+8x+16} &= \frac{2}{x+4} - \frac{3}{(x+4)^2} \\ &= \frac{2(x+4)-3}{(x+4)^2} \\ &= \frac{2x+8-3}{(x+4)^2} \\ &= \frac{2x+5}{(x+4)^2} \end{aligned}$$

**m** $$\begin{aligned} \frac{3}{x-1} + \frac{2}{(x-1)(x+4)} &= \frac{3(x+4)+2}{(x-1)(x+4)} \\ &= \frac{3x+12+2}{(x-1)(x+4)} \\ &= \frac{3x+14}{(x-1)(x+4)} \end{aligned}$$

**n** $$\begin{aligned} \frac{3}{x-2} - \frac{2}{x+2} + \frac{4}{x^2-4} &= \frac{3}{x-2} - \frac{2}{x+2} + \frac{4}{(x-2)(x+2)} \\ &= \frac{3(x+2)-2(x-2)+4}{(x-2)(x+2)} \\ &= \frac{3x+6-2x+4+4}{(x-2)(x+2)} \\ &= \frac{x+14}{(x-2)(x+2)} \end{aligned}$$

**o** $$\begin{aligned} \frac{5}{x-2} + \frac{3}{x^2+5x+6} + \frac{2}{x+3} &= \frac{5}{x-2} + \frac{3}{(x+2)(x+3)} + \frac{2}{x+3} \\ &= \frac{5(x+3)(x+2)+3(x-2)+2(x-2)(x+2)}{(x-2)(x+2)(x+3)} \\ &= \frac{5(x^2+5x+6)+3x-6+2(x^2-4)}{(x-2)(x+2)(x+3)} \\ &= \frac{5x^2+25x+30+3x-6+2x^2-8}{(x-2)(x+2)(x+3)} \\ &= \frac{7x^2+28x+16}{(x-2)(x+2)(x+3)} \end{aligned}$$

**p** $$\begin{aligned} x-y-\frac{1}{x-y} &= \frac{(x-y)(x-y)-1}{x-y} \\ &= \frac{(x-y)^2-1}{x-y} \end{aligned}$$

**q** $$\begin{aligned} \frac{3}{x-1} - \frac{4x}{1-x} &= \frac{3}{x-1} + \frac{4x}{x-1} \\ &= \frac{4x+3}{x-1} \end{aligned}$$

r $$\frac{3}{x-2}+\frac{2}{2-x}=\frac{3}{x-2}-\frac{2x}{x-2}$$
$$=\frac{3-2x}{x-2}$$

2 a $$\frac{x^2}{2y}\times\frac{4y^3}{x}=\frac{4y^3x^2}{2yx}$$
$$=2xy^2$$

b $$\frac{3x^2}{4y}\times\frac{y^2}{6x}=\frac{3x^2y^2}{24yx}$$
$$=\frac{xy}{8}$$

c $$\frac{4x^3}{3}\times\frac{12}{8x^4}=\frac{48x^3}{24x^4}$$
$$=\frac{2}{x}$$

d $$\frac{x^2}{2y}\div\frac{3xy}{6}=\frac{x^2}{2y}\times\frac{6}{3xy}$$
$$=\frac{6x^2}{6xy^2}$$
$$=\frac{x}{y^2}$$

e $$\frac{4-x}{3a}\times\frac{a^2}{4-x}=\frac{a^2(4-x)}{3a(4-x)}$$
$$=\frac{a}{3}$$

f $$\frac{2x+5}{4x^2+10x}=\frac{2x+5}{2x(2x+5)}$$
$$=\frac{1}{2x}$$

g $$\frac{(x-1)^2}{x^2+3x-4}=\frac{(x-1)^2}{(x-1)(x+4)}$$
$$=\frac{x-1}{x+4}$$

h $$\frac{x^2-x-6}{x-3}=\frac{(x-3)(x+2)}{x-3}$$
$$=x+2$$

i $$\frac{x^2-5x+4}{x^2-4x}=\frac{(x-1)(x-4)}{x(x-4)}$$
$$=\frac{x-1}{x}$$

j $$\frac{5a^2}{12b^2}\div\frac{10a}{6b}=\frac{5a^2}{12b^2}\times\frac{6b}{10a}$$
$$=\frac{30a^2b}{120ab^2}$$
$$=\frac{a}{4b}$$

**k** $\dfrac{x-2}{x} \div \dfrac{x^2-4}{2x^2} = \dfrac{x-2}{x} \times \dfrac{2x^2}{x^2-4}$

$= \dfrac{x-2}{x} \times \dfrac{2x^2}{(x-2)(x+2)}$

$= \dfrac{2x^2}{x(x+2)}$

$= \dfrac{2x}{x+2}$

**l** $\dfrac{x+2}{x(x-3)} \div \dfrac{4x+8}{x^2-4x+3} = \dfrac{x+2}{x(x-3)} \div \dfrac{4(x+2)}{(x-1)(x-3)}$

$= \dfrac{x+2}{x(x-3)} \times \dfrac{(x-1)(x-3)}{4(x+2)}$

$= \dfrac{1}{x} \times \dfrac{x-1}{4}$

$= \dfrac{x-1}{4x}$

**m** $\dfrac{2x}{x-1} \div \dfrac{4x^2}{x^2-1} = \dfrac{2x}{x-1} \times \dfrac{x^2-1}{4x^2}$

$= \dfrac{2x}{x-1} \times \dfrac{(x-1)(x+1)}{4x^2}$

$= \dfrac{2x(x+1)}{4x^2}$

$= \dfrac{x+1}{2x}$

**n** $\dfrac{x^2-9}{x+2} \times \dfrac{3x+6}{x-3} \div \dfrac{9}{x} = \dfrac{(x-3)(x+3)}{x+2} \times \dfrac{3(x+2)}{x-3} \times \dfrac{x}{9}$

$= \dfrac{3x(x-3)(x+3)(x+2)}{9(x+2)(x-3)}$

$= \dfrac{x(x+3)}{3}$

**o** $\dfrac{3x}{9x-6} \div \dfrac{6x^2}{x-2} \times \dfrac{2}{x+5} = \dfrac{3x}{3(3x-2)} \times \dfrac{x-2}{6x^2} \times \dfrac{2}{x+5}$

$= \dfrac{2x(x-2)}{6x^2(3x-2)(x+5)}$

$= \dfrac{x-2}{3x(3x-2)(x+5)}$

**3 a** $\dfrac{1}{x-3} + \dfrac{2}{x-3} = \dfrac{3}{x-3}$

**b** $\dfrac{2}{x-4} + \dfrac{2}{x-3} = \dfrac{2(x-3)+2(x-4)}{(x-4)(x-3)}$

$= \dfrac{2x-6+2x-8}{x^2-7x+12}$

$= \dfrac{4x-14}{x^2-7x+12}$

**c**
$$\begin{aligned}\frac{3}{x+4}+\frac{2}{x-3} &= \frac{3(x-3)+2(x+4)}{(x+4)(x-3)}\\ &= \frac{3x-9+2x+8}{x^2+x-12}\\ &= \frac{5x-1}{x^2+x-12}\end{aligned}$$

**d**
$$\begin{aligned}\frac{2x}{x-3}+\frac{2}{x+4} &= \frac{2x(x+4)+2(x-3)}{(x-3)(x+4)}\\ &= \frac{2x^2+8x+2x-6}{x^2+x-12}\\ &= \frac{2x^2+10x-6}{x^2+x-12}\end{aligned}$$

**e**
$$\begin{aligned}\frac{1}{(x-5)^2}+\frac{2}{x-5} &= \frac{1+2(x-5)}{(x-5)^2}\\ &= \frac{1+2x-10}{x^2-10x+25}\\ &= \frac{2x-9}{x^2-10x+25}\end{aligned}$$

**f**
$$\begin{aligned}\frac{3x}{(x-4)^2}+\frac{2}{x-4} &= \frac{3x+2(x-4)}{(x-4)^2}\\ &= \frac{3x+2x-8}{x^2-8x+16}\\ &= \frac{5x-8}{x^2-8x+16}\\ &= \frac{5x-8}{(x-4)^2}\end{aligned}$$

**g**
$$\begin{aligned}\frac{1}{x-3}-\frac{2}{x-3} &= \frac{-1}{x-3}\\ &= \frac{1}{3-x}\end{aligned}$$

**h**
$$\begin{aligned}\frac{2}{x-3}-\frac{5}{x+4} &= \frac{2(x+4)-5(x-3)}{(x-3)(x+4)}\\ &= \frac{2x+8-5x+15}{x^2+x-12}\\ &= \frac{23-3x}{x^2+x-12}\end{aligned}$$

**i**
$$\begin{aligned}\frac{2x}{x-3}+\frac{3x}{x+3} &= \frac{2x(x+3)+3x(x-3)}{(x-3)(x+3)}\\ &= \frac{2x^2+6x+3x^2-9x}{x^2-9}\\ &= \frac{5x^2-3x}{x^2-9}\end{aligned}$$

**j**
$$\begin{aligned}\frac{1}{(x-5)^2}-\frac{2}{x-5} &= \frac{1-2(x-5)}{(x-5)^2}\\ &= \frac{1-2x+10}{x^2-10x+25}\\ &= \frac{11-2x}{x^2-10x+25}\end{aligned}$$

**k**
$$\begin{aligned}\frac{2x}{(x-6)^3}-\frac{2}{(x-6)^2} &= \frac{2x-2(x-6)}{(x-6)^3}\\ &= \frac{2x-2x+12}{(x-6)^3}\\ &= \frac{12}{(x-6)^3}\end{aligned}$$

**l**
$$\begin{aligned}\frac{2x+3}{x-4}-\frac{2x-4}{x-3} &= \frac{(2x+3)(x-3)-(2x-4)(x-4)}{(x-4)(x-3)}\\ &= \frac{(2x^2-3x-9)-(2x^2-12x+16)}{x^2-7x+12}\\ &= \frac{2x^2-3x-9-2x^2+12x-16}{x^2-7x+12}\\ &= \frac{9x-25}{x^2-7x+12}\end{aligned}$$

**4 a**
$$\begin{aligned}\sqrt{1-x}+\frac{2}{\sqrt{1-x}} &= \frac{\sqrt{1-x}\sqrt{1-x}+2}{\sqrt{1-x}}\\ &= \frac{1-x+2}{\sqrt{1-x}}\\ &= \frac{3-x}{\sqrt{1-x}}\end{aligned}$$

**b**
$$\frac{2}{\sqrt{x-4}}+\frac{2}{3} = \frac{2\sqrt{x-4}+6}{3\sqrt{x-4}}$$

**c**
$$\frac{3}{\sqrt{x+4}}+\frac{2}{\sqrt{x+4}} = \frac{5}{\sqrt{x+4}}$$

**d**
$$\begin{aligned}\frac{3}{\sqrt{x+4}}+\sqrt{x+4} &= \frac{3+\sqrt{x+4}\sqrt{x+4}}{\sqrt{x+4}}\\ &= \frac{3+x+4}{\sqrt{x+4}}\\ &= \frac{x+7}{\sqrt{x+4}}\end{aligned}$$

**e**
$$\begin{aligned}\frac{3x^3}{\sqrt{x+4}}-3x^2\sqrt{x+4} &= \frac{3x^3-3x^2\sqrt{x+4}\sqrt{x+4}}{\sqrt{x+4}}\\ &= \frac{3x^3-3x^2(x+4)}{\sqrt{x+4}}\\ &= \frac{3x^3-3x^3-12x^2}{\sqrt{x+4}}\\ &= -\frac{12x^2}{\sqrt{x+4}}\end{aligned}$$

**f** $\dfrac{3x^3}{2\sqrt{x+3}} + 3x^2\sqrt{x+3} = \dfrac{3x^3 + 6x^2\sqrt{x+3}\sqrt{x+3}}{2\sqrt{x+3}}$

$= \dfrac{3x^3 + 6x^2(x+3)}{2\sqrt{x+3}}$

$= \dfrac{3x^3 + 6x^3 + 18x^2}{2\sqrt{x+3}}$

$= \dfrac{9x^3 + 18x^2}{2\sqrt{x+3}}$

$= \dfrac{9x^2(x+2)}{2\sqrt{x+3}}$

**5 a** $(6x-3)^{\frac{1}{3}} - (6x-3)^{-\frac{2}{3}} = (6x-3)^{\frac{1}{3}} - \dfrac{1}{(6x-3)^{\frac{2}{3}}}$

$= \dfrac{(6x-3)^{\frac{1}{3}}(6x-3)^{\frac{2}{3}} - 1}{(6x-3)^{\frac{2}{3}}}$

$= \dfrac{6x-3-1}{(6x-3)^{\frac{2}{3}}}$

$= \dfrac{6x-4}{(6x-3)^{\frac{2}{3}}}$

**b** $(2x+3)^{\frac{1}{3}} - 2x(2x+3)^{-\frac{2}{3}} = (2x+3)^{\frac{1}{3}} - \dfrac{2x}{(2x+3)^{\frac{2}{3}}}$

$= \dfrac{(2x+3)^{\frac{1}{3}}(2x+3)^{\frac{2}{3}} - 2x}{(2x+3)^{\frac{2}{3}}}$

$= \dfrac{2x+3-2x}{(2x+3)^{\frac{2}{3}}}$

$= \dfrac{3}{(2x+3)^{\frac{2}{3}}}$

**c** $(3-x)^{\frac{1}{3}} - 2x(3-x)^{-\frac{2}{3}} = (3-x)^{\frac{1}{3}} - \dfrac{2x}{(3-x)^{\frac{2}{3}}}$

$= \dfrac{(3-x)^{\frac{1}{3}}(3-x)^{\frac{2}{3}} - 2x}{(3-x)^{\frac{2}{3}}}$

$= \dfrac{3-x-2x}{(3-x)^{\frac{2}{3}}}$

$= \dfrac{3-3x}{(3-x)^{\frac{2}{3}}}$

Since $(3-x)^2 = (x-3)^2$, the answer is equivalent to $\dfrac{3-3x}{(x-3)^{\frac{2}{3}}}$.